Analysis of unsteady waves in solids*

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(Received 15 June 1972)

In the present paper the acceleration wave theory applicable to the analysis of unsteady waves is developed. It is noted that a measurement of either the stress or the velocity history at a material point is sufficient to determine the history of the remaining one of these variables and of the strain if the instantaneous sound speed is known. This sound speed can be approximated by the speed determined as the wave passes through the material between closely spaced gauge stations or may be directly calculated from simultaneous measurements of particle velocity and stress at a single station. The present theory permits the analysis of wave-profile data obtained using conventional instrumentation which has a time resolution of a few nanoseconds.

I. INTRODUCTION

When a solid is subjected to the rapid impulsive loading accompanying impact or explosion, waves propagate with amplitudes and speeds which are governed by the mechanical properties of the sample. After suitable analysis, experimental measurements of the speeds and amplitudes of these waves are frequently utilized to obtain a description of the constitutive relation of the shock-loaded sample. In the earliest investigations, experimental data were obtained only for pressures well above 100 kbar and were analyzed with conservation relations which assumed that compressive waves propagated as steady shocks. At high pressure this assumption is valid to within the limits of available instrumentation to make time-resolved measurements. More recent work in the pressure range of approximately 1—100 kbar has shown that compression waves may be unsteady, frequently exhibit appreciable rise times, and often possess a very complex structure.

While the theoretical and experimental bases for high-pressure shock-wave studies are well established and widely employed,[1–3] analysis of experiments in which recorded waveforms exhibit finite rise times is more complicated and less certain. In the simplest instance, these structured waves consist simply of two or more shocks and can be interpreted in terms of the shock jump conditions with little difficulty. Observed waveforms involving smooth transitions between states have been analyzed by approximating the observed stress or particle-velocity history as a sequence of shocks so that the approximate waveform can be analyzed by means of the jump conditions for shocks.[4–6] Even though this procedure for treating unsteady waveforms seems to provide a suitable data-reduction technique in most instances, the extent of the approximations has not been defined and no explicit theoretical basis for the analysis has been offered. A well-founded and effective scheme for reducing the data obtained in conventional shock-compression experiments conducted in the low-pressure regime is urgently needed. Since there is no universally acceptable constitutive equation for describing the nonequilibrium behavior of most of the various classes of materials that propagate waves exhibiting appreciable structure, it is desirable that the data-reduction scheme be one in which information can be extracted without the assumption of any such relation.

A recent examination of this problem by Fowles and Williams[7] has given rise to a new theory of unsteady wave propagation. This theory involves different speeds of propagation of stress and particle-velocity waves; hence, it will be referred to as the dual-wave theory. Unfortunately, attempts to base a data-reduction scheme on the dual-wave theory have been frustrated by the necessity to employ instrumentation of a type that, in its present state of development, does not exhibit sufficiently good time resolution to study many materials of interest.

In this paper an alternative scheme is presented which, while not directly meeting the challenge posed by the dual-wave theory, has the advantage of providing a rational and explicit method for reducing data obtained from conventional instrumentation. This method is similar to that in which the wave is approximated as a sequence of steps (shocks) but employs the higher approximation in which the experimental record is approximated as a sequence of chords. The discontinuities of slope where the chords join form acceleration waves, and the data-reduction method proposed in this paper is based on the theory of these waves. Only a single wave speed (the acceleration wave speed) appears in the governing equations, which are otherwise similar to those of the dual-wave theory. The proposed data-reduction scheme occupies an intermediate position between the conventional method of approximating the record as a sequence of shocks and the dual-wave method of Fowles and Williams. It is better founded than the former method, and yet still preserves its essential advantages of experimental simplicity and ability to cope with wave interactions.

After considering kinematical preliminaries in Sec. II and pertinent results from the theory of singular surfaces in Sec. III, the properties of acceleration waves will be considered in Sec. IV. In Sec. V, the analysis will be specialized to the case of one-dimensional motions, and in the discussion of Sec. VI the acceleration wave theory will be compared to other results. In the Appendix it is shown that the analysis can be employed to solve wave interaction problems.

II. KINEMATICAL PRELIMINARIES

Let $\mathbf{X}=(X^1,X^2,X^3)$ denote the coordinates of a material point at time t_0, and let $\mathbf{x}=(x^1,x^2,x^3)$ denote the coordinates of the same material point at time $t\geq t_0$. A motion of a material body is described by the function $\hat{\mathbf{x}}$ such that

$$\mathbf{x}=\hat{\mathbf{x}}(\mathbf{X},t), \tag{2.1}$$

with the property $\hat{\mathbf{x}}(X,t_0)=\mathbf{X}$. The physical interpretation of the function $\hat{\mathbf{x}}$ is that it gives the coordinates $\mathbf{x}$ of a material point at each time $t>t_0$, whose position at $t=t_0$ is given by the coordinates $\mathbf{X}$. In the treatment of physical problems, the usage of either $(\mathbf{X},t)$ or $(\mathbf{x},t)$ as independent variables is equivalent. $(\mathbf{X},t)$ is referred to as the material description while $(\mathbf{x},t)$ is referred to as the spatial description. In this paper we employ the material description, because we have in mind application to problems of solid mechanics in which the instrumentation is affixed to a specific particle and records the history of events taking place at this particle.

The components $\dot{x}^k$ of the velocity $\dot{\mathbf{x}}$ of a particle and the components $\ddot{x}^k$ of the acceleration $\ddot{\mathbf{x}}$ of a particle are given by

$$\dot{x}^k=\frac{\partial\hat{x}^k(\mathbf{X},t)}{\partial t},\quad \ddot{x}^k=\frac{\partial^2\hat{x}^k(\mathbf{X},t)}{\partial t^2}. \tag{2.2}$$

At each $t>t_0$, the components $x^k_{,K}$, $x^k_{,KL}$ of the deformation gradient and its derivative, respectively, are given by

$$x^k_{,K}=\frac{\partial\hat{x}^k(\mathbf{X},t)}{\partial X^K}\quad\text{and}\quad x^k_{,KL}=\frac{\partial\hat{x}^k(\mathbf{X},t)}{\partial X^K\,\partial X^L}. \tag{2.3}$$

III. RESULTS FROM THE THEORY OF SINGULAR SURFACES

The theory of singular surfaces and the conditions that must be satisfied across a singular surface for geometrical and for kinematical reasons have been presented, for instance, by Truesdell and Toupin.[8] Here only the main features of singular surface analysis will be reviewed and applied to the problem at hand.

Consider a material region R and a surface S that divides the region into R^+ and R^-. The unit normal $\mathbf{N}$ to the surface S is directed toward R^+. Let $\Psi(\cdot,t)$ be a function which is continuous within the regions R^+ and R^- at each $t>t_0$ and for which the limits Ψ^+ and Ψ^- exist as $\mathbf{X}$ approaches a point $\mathbf{X}_0$ on S along paths wholly within R^+ and R^-. We say that the surface S is singular with respect to $\Psi(\mathbf{X},t)$ if

$$[\Psi]\equiv\Psi^+-\Psi^-\neq 0.$$

If the surface S is a moving surface, i.e., a wave, then it is necessary to discuss the manner in which it may move. Basically, there are three ways of describing speeds by which this can be accomplished. (i) u_n—the speed of displacement. It is a measure of the speed with which the surface moves with respect to the origin of our fixed rectangular Cartesian system. (ii) U_N—the speed of propagation. It is a measure of the speed with which the surface traverses the material. (iii) U—the local speed of propagation. It is a measure of the speed with which the surface moves with respect to the particles instantaneously on the surface, i.e., $U=u_n-\dot{x}_n$. $\dot{x}_n$ is the normal component of the velocity of the particles instantaneously on the surface with respect to the spatial direction $\mathbf{n}$ in which the surface is moving.

In the present discussion, the following definitions are needed:

(i) A wave is said to be a shock wave if the conditions

$$[x^k]=0,\quad [\dot{x}^k]\neq 0,\quad\text{and}\quad [x^k_{,K}]\neq 0 \tag{3.1}$$

are satisfied.

(ii) A wave is said to be an acceleration wave if

$$\begin{aligned}&[x^k]=[\dot{x}^k]=[x^k_{,K}]=0,\quad [\ddot{x}^k]\neq 0,\\ &[\dot{x}^k_{,K}]\neq 0,\quad\text{and}\quad [x^k_{,KM}]\neq 0.\end{aligned} \tag{3.2}$$

Higher-order waves can be defined in an analogous manner.

A singular surface is called a contact surface if $[\dot{\mathbf{x}}]=0$ and $U=0$. A contact surface has no motion with respect to the material but may, of course, be convected along with the material.

The requirements of conservation of mass and conservation of linear momentum across a singular surface are of the form

$$[\rho U]=0, \tag{3.3}$$

and

$$[t^{km}]n_m+[\rho U\dot{x}^k]=0, \tag{3.4}$$

where t^{km} is called the Cauchy stress or the true stress, and ρ is the present density.

It should be pointed out that relations (3.3) and (3.4) are arrived at independent of the nature of the surface. These are the relations which must be satisfied for every wave or contact surface if mass and linear momentum are to be conserved across the surface. For the case of shock waves when values of U and $[\dot{x}^k]$ are known, these relations are used to determine the value of the density and the value of the stress components behind the shocks. Furthermore, these relations are arrived at independent of the material in question, i.e., independent of the constitutive relations.